An Application of Differential Equations to Political Partisanship

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Abstract

The current political climate suggests an overarching decrease in political partisanship in the United States. In order to test this theory, we will consider a differential equations model suited for two-group competition theory. We will first consider the model generally, addressing the stability of all possible equilibrium points that could be predicted. Then, using data on United States political partisanship versus non-partisanship, we find parameters for the model that form a best fit for the data. Finally, we use the model to project the trend towards or away from party identity, and determine whether it seems as though an eventual steady state will emerge.

1 Introduction

Political partisanship is of the utmost importance to political analysts and politicians simply because it impacts the outcomes of elections. Partisanship is a well-established phenomenon, and has existed essentially since the formation of the United States. However, there has been concern since the 1960s and 1970s that partisanship is in decline, as more people begin to identify themselves as "independent" from a party [1]. There has also been an increasing dissatisfaction with the two-party system since the 1980s [4]. In looking at the political climate at the time of this paper, there is also great dissatisfaction with both major political parties: the Democrats and the Republicans. This tension was especially high during the recent government shutdown, which occurred from October 1 to October 16, 2013. In fact, according to a Gallup poll, Congressional approval reached a record low of 9% in November of 2013 [2]. People are becoming increasingly frustrated with government's seeming inability to pass legislation due to partisan conflicts. Thus, we can infer that the proportion of United States citizens identifying as belonging to a political party should be decreasing. This investigation uses a differential equations model to predict future trends in political partisanship in the United States. We will use actual data to find the best parameters for our model, then look at how the solution projects changes in partisanship in the future.

The model used here was developed by Dr. Haley Yaple and Dr. Danny Abrams for Dr. Yaple's doctoral thesis at Northwestern University. In this investigation, we expand upon their work via this new application to political partisanship. We also expand on the general analysis of the model by considering all possible future steady states based on differing values for the parameters.

2 Definitions and Development

We will now define some terms necessary to the understanding of the model and results. We will assume a basic understanding of differential equations, instead focusing on more complex and specific definitions relevant to this particular model.

We will use a model appropriate for two-group competition, which is defined here.

Definition 1 Two-group competition means that there are exactly two competing groups within a population.

We will refer to these groups as X and Y. There are several conditions that must be satisfied in order to use the model. 1) The two groups must be mutually exclusive. 2) The entire population must belong to one of the two groups. 3) The population must be approximately static in size. If it is not, then we must be able to assume that neither group is increasing in size faster than the other. 4) It must be easy to move between the two groups.

If these four conditions are met, then we are able to use a fairly simple differential equations model to look at the movement between the two groups. In order to develop this model, we need several more definitions. The model takes two major components into account when determining the movement between groups. The first is the majority effect.

Definition 2 The **majority effect** is the assumption that there is a higher incentive for members of the population to belong to the group that is in the majority.

The second major component is the utility of belonging to a group.

Definition 3 Utility refers to the overall benefit of belonging to a group for any reason besides the majority effect.

The utility of belonging to group X will be referred to as u_x. We will also assume that the utility of belonging to groups X and Y are inverse, so that as u_x increases, u_y decreases. As such, we can define these variables so that $(u_x + u_y) = 1$.

We will use another parameter, a, to vary the importance of the majority effect relative to the utility for movement between the groups. Our equation will use a as an exponent on the proportion belonging to each group at time t, such that if a is greater than 1, the overall effect of the proportion belonging to the group will decrease, making the majority effect a smaller force than the utility. If a is less than 1, however, the overall impact of the proportion belonging to the group will be greater, giving the majority effect more importance than the utility. We will assume that a is always greater than 0.

We will also use a time parameter, c, to vary the time span of the model to fit the data. If c is small, this means that our data points are relatively close in time, whereas the opposite is true if c is large.

Our model is a differential equation that takes these parameters into account in order to show the change in the proportion belonging to a single group at time t. Specifically, we will model the change in x, the proportion of the population belonging to group X. Thus y, the proportion belonging to group Y, is simply $(1 - x)$ since the two groups encompass the entire population.

Our model is

$$\frac{dx}{dt} = (c)(u_x)(x^a)(1-x) - (c)(1-u_x)(1-x)^a(x) \quad (1).$$

This model looks at the change in *x* at time *t* in two pieces. The first piece reflects the movement into group *X* from group *Y*, while the second piece shows the movement out of group *X* into group *Y*. Because of the majority effect, we expect the movement in to be directly proportional to x^a. The movement out, then, is directly proportional to y^a, which is simply $(1 - x)^a$. We also expect the movement into group *X* to be directly proportional to u_x and the movement out to be directly proportional to u_y. Again, because of the way we have defined utility, we substitute $(1 - u_x)$ for u_y. Parameter *a*, as discussed previously, is an exponent on both *x* and *(1 - x)* in order to vary the importance of the majority effect relative to the utility. Finally, our time parameter *c* is directly proportional to both the movement in and the movement out of group *X*.

We will also, in the course of this study, consider the long-term trend of the model. In order to do so, we will first define some mathematical terms that will help us determine what our data projects for the future. We will find that the model can help us determine future equilibrium points based on our data. The following definitions are intuitive rather than rigorous; however, they serve the needs of this study.

Definition 4 An **equilibrium point** in differential equations refers to any point where the derivative with respect to time is equal to 0. This means that there is no movement at that point.

There are two types of equilibrium points. The first is a stable equilibrium point.

Definition 5 A **stable equilibrium point** attracts. This means that if the solution is at a point close to the equilibrium point, it tends to move towards that equilibrium point.

The second type of equilibrium point is unstable.

Definition 6 An **unstable equilibrium point** repels. This means that if we are near an unstable equilibrium point, we will tend to move away from that point.

We will be looking at changes in political partisanship in the United States. For the sake of this study, we will operationally define *partisanship* to mean self-identification as either “Democrat” or “Republican”. We operationally define *non-partisanship* to mean anything else, including those who identify as “Independent” or as none of these. We will consider the proportion of the population identifying with a party to be represented by the variable *x*.

The data used for this study was taken from Gallup, a research company that is well known for its public opinion polls, especially with regard to politics. The data is given as percentages rounded to the nearest whole number, which was then converted to two-decimal proportions for the sake of this study. Each data point appears to have been collected over a short time interval (approximately two to three days). For simplicity, the data was restructured to refer to only the last day of each time interval, giving a single data point. The time between samples ranges from two to fifty-five days, and dates back to January 5, 2004. As such, it is convenient to refer to the time component of each data point in number of days since January 1, 2004. Figure 1 gives a sample of this data.

Days since 1/1/2004	Proportion Partisan	Proportion Nonpartisan
4	.60	.40
10	.64	.36
14	.66	.34
31	.64	.36
…	…	…

Figure 1: Partisanship Data Table Sample

3 Results

We will first discuss results regarding the model generally, then move into the application to political partisanship trends.

Without loss of generality, we can consider a variation on our model that disregards the *c* parameter, such that the equation is

$$\frac{dx}{dt} = (u_x)(x^a)(1-x) - (1-u_x)(1-x)^a(x).$$

We find that we can factor *x* and *(1-x)* out of the equation, such that

$$\frac{dx}{dt} = (x)(1-x)[(u_x)\left(x^{(a-1)}\right) - (1-u_x)(1-x)^{(a-1)}] \quad (2).$$

From here, we will move into a discussion of the equilibrium points of the model. We will assume, for all cases, that $0 < u_x < 1$. If $u_x = 0$ or $u_x = 1$, it is easy to see that the solution to our differential equation would decrease to $x = 0$ or increase to $x = 1$, respectively. However, we will address all other cases.

Theorem 1 (Equilibrium Points Theorem) For any $0 < u_x < 1$ and with $a > 0$, $a \neq 1$, there are exactly three equilibrium points for equation (2) on the domain $0 \leq x \leq 1$.

Proof. To find the equilibrium points of our model, we set equation (2) equal to 0. With the factored version of the equation, it is easy to see that $x = 0$ and $x = 1$ are equilibrium points. If we set the remaining term equal to 0, we see that there is a third equilibrium point dependent on the values of u_x and *a*, such that

$$(u_x)x^{(a-1)} - (1-u_x)(1-x)^{(a-1)} = 0.$$

This equation is easily solved in terms of u_x. Thus, we can write a function for the third equilibrium point in terms of u_x, such that

$$u_x = \frac{1}{1+(1-x)^{(1-a)}x^{(a-1)}} \quad (3).$$

To prove that this results in at most one final equilibrium point, it is sufficient to prove that function (3) is either always increasing or always decreasing, implying that it is one-to-one. Then, to show that there is at least one equilibrium point, it is enough to show that the limits of function (3) as x approaches 0 and 1 are either 0 or 1. This would mean that function (3) results in exactly one equilibrium point for every paired $0 < u_x < 1$ and $a > 0$, $a \neq 1$.

We will prove Theorem 1 in two cases.

First, consider the case where $a < 1$. To show that the function is either always increasing or always decreasing, we first take the partial derivative of equation (3) with respect to x and find that

$$\frac{\partial u_x}{\partial x} = -\frac{(a-1)(1-x)^a x^a}{x(1-x)^a + (1-x)x^a} \quad (4).$$

We will substitute $k = (1 - a)$, where $0 < k < 1$. Now, our equation simplifies to

$$\frac{\partial u_x}{\partial x} = \frac{(k)(1-x)^a x^a}{x(1-x)^a + (1-x)x^a}.$$

Due to our assumptions about a and x, we see that the numerator is positive because all of its terms are positive. Similarly, we see that the denominator is positive. Thus, for $a < 1$, function (3), is always increasing, which implies that there is at most one solution for x for every paired $0 < a < 1$ and $0 < u_x < 1$.

Next, we will return to function (3):

$$u_x = \frac{1}{1+(1-x)^{(1-a)}x^{(a-1)}}.$$

Since we are assuming $0 < a < 1$, we will substitute $k = (1 - a)$, where $0 < k < 1$. Then our equation becomes

$$u_x = \frac{x^k}{1+(1-x)^k}.$$

It is easy to see that $\lim_{x\to 0}\left(\frac{x^k}{1+(1-x)^k}\right) = 0$, and that $\lim_{x\to 1}\left(\frac{x^k}{1+(1-x)^k}\right) = 1$. This implies that there is at least one solution for x for every paired $0 < a < 1$ and $0 < u_x < 1$. Thus, there must be exactly one solution for x within these parameters.

Second, we will consider the case where $a > 1$. Similarly, we will substitute $k = (a - 1)$, $0 < k$ into equation (4), and find

$$\frac{\partial u_x}{\partial x} = -\frac{(k)(1-x)^a x^a}{x(1-x)^a + (1-x)x^a}.$$

Now, we see that the numerator of the equation is negative. The denominator of the equation remains the same, and is again positive by the same reasoning as above. Thus, for $a > 1$, our function for u_x given by equation (3) is always decreasing, implying that there is at most one solution for x for every paired a and u_x.

Now, we return again to function (3):

$$u_x = \frac{1}{1+(1-x)^{(1-a)}x^{(a-1)}}.$$

Since we are now assuming $1 < a$, we substitute $k = (a - 1)$, where $0 < k$. Then our function becomes

$$u_x = \frac{(1-x)^k}{1+x^k}.$$

It is easy to see that $\lim_{x\to 0}\left(\frac{(1-x)^k}{1+x^k}\right) = 1$ and $\lim_{x\to 1}\left(\frac{(1-x)^k}{1+x^k}\right) = 0$. This shows that there is at least one solution for *x* for every paired 1 < *a* and 0 < u_x < 1.Thus, there is exactly one solution for *x* within these parameters.

We have now shown that there are exactly three equilibrium points for equation (2) for any given 0 < u_x < 1 and *a* > 0, *a* ≠ 1, on the domain 0 ≤ *x* ≤ 1. The first is *x* = 1, the second is *x* = 0. The third depends on both u_x and *a*, and can be found in each particular case using equation (3). ■

Now that we have determined the possible equilibrium points, we will consider their stability. In order to do this, we will utilize an established theorem, stated here.

Theorem 2 Let *f(y)* be a differentiable function on an interval *I*, and suppose that *f(p) = 0*. Let *K* = $f_y(p)$. If *K < 0*, then *y ≡ p* is an attracting equilibrium point for the differential equation *y' = f(y)*. Similarly, *y ≡ p* is a repelling equilibrium point for the differential equation *y' = f(y)* if *K > 0*. If *K = 0*, the equilibrium point may be attracting, repelling, or neither [3].

Using Theorem 2, we can establish two lemmas regarding the stability of the equilibrium points *x* = 1 and *x* = 0.

Lemma 1 If *a* > 1, then equation (2) has stable equilibrium points at *x* = 1 and *x* = 0 for 0 < u_x < 1.

Proof. In order to use Theorem 2, we take *f(x)* to be equation (2). From here, we find that:

$$f_x(x) = \frac{(1-u_x)(1-(1+a)(x))}{(1-x)^{(1-a)}} + \frac{(u_x)((1-x)(a)-x)}{x^{(1-a)}}.$$

Plugging in our equilibrium points, we find that:

$$f_x(1) = 0^{(a-1)}(a)(1-u_x) - u_x$$

and

$$f_x(0) = 0^{(a-1)}(a)(u_x) + u_x - 1.$$

Because *a* > 1, these equations simplify to

$$f_x(1) = -u_x$$

and

$$f_x(0) = u_x - 1.$$

Since 0 < u_x < 1, $f_x(1)$ and $f_x(0)$ are negative for all values of u_x. By Theorem 2, these equilibrium points are both stable. ■

We will use the same reasoning in the following lemma, however in this lemma we will deal with the case where 0 < *a* < 1.

Lemma 2 If $0 < a < 1$, then equation (2) has unstable equilibrium points at $x = 1$ and $x = 0$ for $0 < u_x < 1$.

Proof. By the same reasoning as used to prove Lemma 1, we find that

$$f_x(1) = 0^{(a-1)}(a)(1-u_x) - u_x$$

and

$$f_x(0) = 0^{(a-1)}(a)(u_x) + u_x - 1.$$

Because $a < 1$, we see that this results in division by 0. So, we will instead look at how the partial derivative behaves as x approaches both 1 and 0. We will consider our original partial derivative,

$$f_x(x) = \frac{(1-u_x)(1-(1+a)(x))}{(1-x)^{(1-a)}} + \frac{(u_x)((1-x)(a)-x)}{x^{(1-a)}},$$

for each case separately.

We can see that the limit of $f_x(x)$ goes to positive infinity as x approaches 1, because the first term approaches positive infinity while the second term of the equation becomes small. Thus, $f_x(1)$ is positive, implying that $x = 1$ is an unstable equilibrium point by Theorem 2**.**

Now, consider the limit of $f_x(x)$ as x approaches 0. We can see that this limit, too, approaches positive infinity, because the first term in the equation approaches something small, while the second term of the equation approaches positive infinity. This implies that $x = 0$ is also an unstable equilibrium point by Theorem 2. ■

Now that we have specified the stabilities of $x = 1$ and $x = 0$ for the cases where $1 < a$ and $0 < a < 1$, we will now separately consider the case when $a = 1$.

Lemma 3 If $a = 1$ and $0 < u_x < 1$, $u_x \neq 0.5$, then there are exactly two equilibrium points for equation (2): $x = 1$ and $x = 0$. If $u_x < 0.5$, then $x = 1$ is unstable and $x = 0$ is stable. If $u_x > 0.5$, then $x = 1$ is stable and $x = 0$ is unstable.

Proof. In the case where $a = 1$, our factored equation (2) simplifies to

$$\frac{dx}{dt} = (x)(1-x)(2u_x - 1).$$

It is easy to see that $x = 1$ and $x = 0$ are still equilibrium points in this case. However, the third term of this equation no longer has an x term in it, such that if $u_x = 0.5$, the entire equation is equal to 0, meaning that there is no movement between groups in this case. If $u_x \neq 0.5$, it is easy to see that the solution to the differential equation is always either increasing or decreasing when x is not equal to 1 or 0. Whether it is increasing or decreasing depends strictly on the value of u_x. If $u_x < 0.5$, then $\frac{dx}{dt}$ is positive on the domain (0,1), and $x = 1$ is stable, while $x = 0$ is unstable. If $u_x > 0.5$, then then $\frac{dx}{dt}$ is negative, implying that $x = 1$ is unstable and $x = 0$ is stable.■

We now have a full classification for the stability of the equilibrium points $x = 1$ and $x = 0$ for all values of a and u_x. They are presented here in Figure 2.

	$u_x < 0.5$		$u_x > 0.5$		$u_x = 0.5$	
	***x* = 1**	***x* = 0**	***x* = 1**	***x* = 0**	***x* = 1**	***x* = 0**
***a* > 1**	Stable	Stable	Stable	Stable	Stable	Stable
0 < *a* < 1	Unstable	Unstable	Unstable	Unstable	Unstable	Unstable
***a* = 1**	Unstable	Stable	Stable	Unstable	Undefined	Undefined

Figure 2: Equilibrium Point Stability Table for $x = 1$ and $x = 0$

By Theorem 1, we also know that, for $0 < a$, $a \neq 0$, there is a third equilibrium point that can be found using function (3).

We will now use what we know about the stability of the equilibrium points at $x = 1$ and $x = 0$ to determine the stability of the intermediate equilibrium point for $0 < a$, $a \neq 1$.

Theorem 3 (Intermediate Equilibrium Point Stability Theorem) For equation (2), the intermediate equilibrium point $0 < x < 1$ is unstable if $a > 1$, and is stable if $0 < a < 1$.

Proof. We will consider this proof in two cases. First, assume $a > 1$. We have shown in Theorem 1 that there are only three equilibrium points on the domain $0 \leq x \leq 1$. Two of these equilibrium points are $x = 1$ and $x = 0$. The third equilibrium point must then occur between these points, because these points are the extremities of our domain. We also showed in Lemma 1 that $x = 1$ and $x = 0$ are stable equilibrium points. Thus, by definition, the differential equation must be negative (trending towards $x = 0$) at some point near $x = 0$, and must be positive (trending towards $x = 1$) at some point near $x = 1$. Because our function is continuous, we then know by the Intermediate Value Theorem and by Theorem 1 that the equation must pass through the x-axis at the third equilibrium point [5]. This means that at all points to the left of the intermediate equilibrium point, the equation is trending away from this equilibrium point. Similarly, to the right of the equilibrium point, all points are trending away from the equilibrium point in the positive direction. It is beneficial to depict this relationship graphically, shown here in Figure 3.

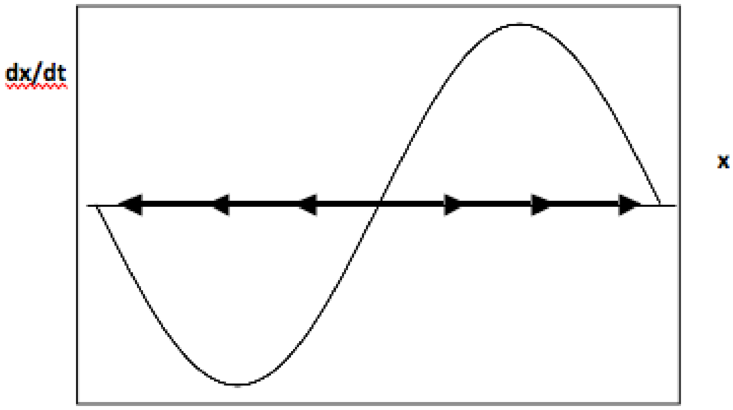

Figure 3: Equilibrium Point Stability for $a > 1$

Thus, the intermediate equilibrium point in the case where $a > 1$ is, by definition, unstable.

Similarly, we will consider the case where 0 < *a* < 1. In this scenario, *x* = 1 and *x* = 0 are both unstable. Thus, at a point near *x* = 0, the differential equation must be positive (trending away from *x* = 0), and it must be negative at a point near *x* = 1 (trending away from *x* = 1). Following the same reasoning as above, this means that for all points to the left of the intermediate equilibrium point the equation is positive (trending towards the equilibrium point), whereas to the right of the equilibrium point the equation is negative (trending towards the equilibrium point). We depict this scenario graphically in Figure 4.

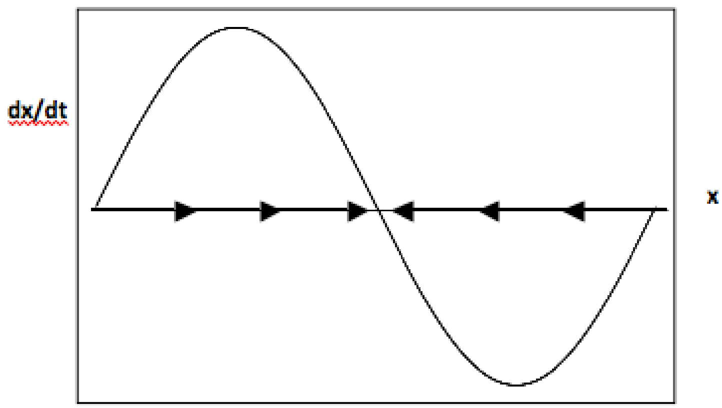

Figure 4: Equilibrium Point Stability for 0 < *a* < 1

By definition, we then know that the intermediate equilibrium point in the case where 0 < *a* < 1 is stable. ■

We have now classified the stability of all possible equilibrium points for equation (2). They are shown here in Figure 5.

	$u_x < 0.5$			$u_x > 0.5$			$u_x = 0.5$		
	***x* = 1**	**Interm.**	***x* = 0**	***x* = 1**	**Interm.**	***x* = 0**	***x* = 1**	**Interm.**	***x* = 0**
***a* > 1**	Stable	Unstable	Stable	Stable	Unstable	Stable	Stable	Unstable	Stable
0 < *a* < 1	Unstable	Stable	Unstable	Unstable	Stable	Unstable	Unstable	Stable	Unstable
***a* = 1**	Unstable	N/A	Stable	Stable	N/A	Unstable	Undefined	Undefined	Undefined

Figure 5: Equilibrium Point Stability Table

Now we will consider results specifically applying to data on political partisanship. We will revert back to equation (1) for our model, because it is now appropriate to utilize the time variable *c*. We now need to find parameters to fit the solution for equation (1) to our Gallup poll data. In looking at the full data set, as depicted in Figure 6, we see that there is an interesting trend that emerges.

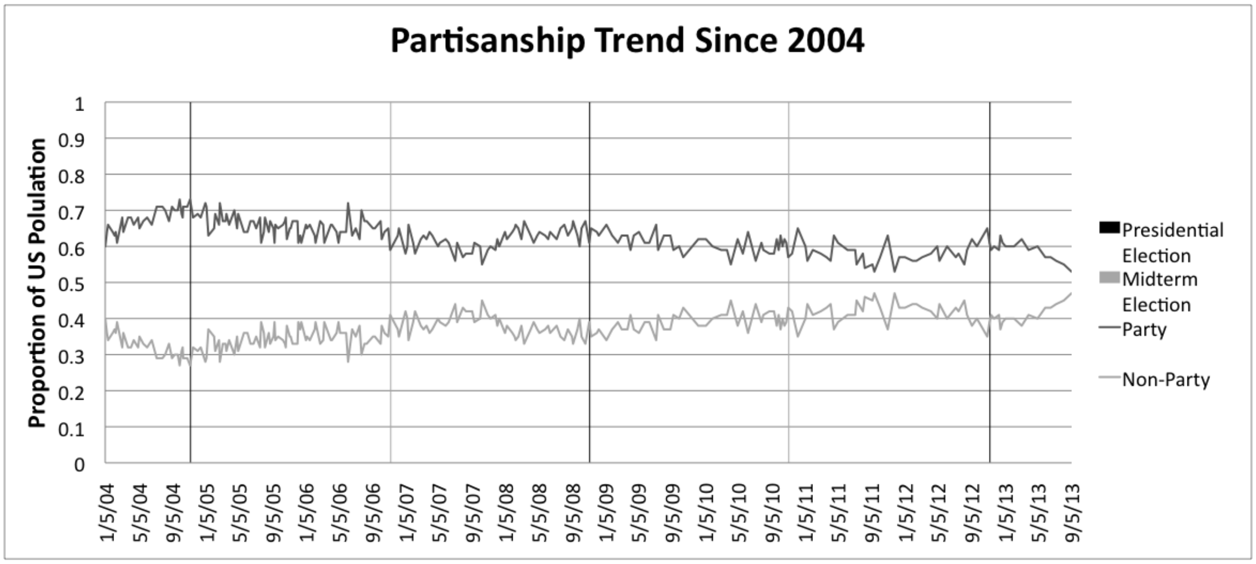

Figure 6: Partisanship Trend Since 2004

We can see in Figure 6, prior to each presidential election (the dark vertical bars) there is a noticeable increase in partisanship. Then, after each election, there is a decline in partisanship. The same is not true, however, regarding midterm elections (the light vertical bars). Thus, it appears as though something is changing pre- and post- presidential election. In order to address this, we will consider three data sets. First, we will consider the data set as a whole. Second, we will address data beginning in 2012 leading up to the 2012 presidential election. Finally, we will consider data following the 2012 election and ending in September of 2013 (the most current data available at the time of this study).

To do this, we will find the values for u_x, a, c, and an initial condition that produce the least squared error when the equation is applied to our data. Our differential equations model cannot be solved directly without values for the parameters, however. So, we will instead iterate through feasible values for the parameters with a relatively small step size. With each iteration, we will solve the differential equation for the specified parameters, apply the result to our data, and calculate the squared error. This is accomplished in Mathematica, where the NDSolve command is utilized to find an interpolating function as a solution for the given differential equation. We will now consider each data set in turn.

In determining the range of values for each parameter, we will use some narrowing assumptions. First, we will allow u_x to vary between 0 and 1 by definition. Second, we will allow the initial condition to vary between the smallest and largest values of x found in each data set. For the remaining parameters, a and c, we have determined via trial and error with larger step sizes a range that seems feasible. In all cases, we have taken a step size of 0.01, such that there are approximately 2 to 3 million iterations for each data set.

First, consider the entire data set. After several trials, we narrow down the ranges for our parameters, and find that the least squared error is 0.217681. The ranges for each parameter and the values that produced the least squared error are presented in Figure 7.

Parameter	Lower Limit	Upper Limit	Final Value	Step Size
a	0.75	1.25	0.95	0.01
$\boldsymbol{u_x}$	0	1	0.50	0.01
c	0	0.25	0.01	0.01
Initial condition	0.53	0.73	0.68	0.01

Figure 7: Parameters for Full Data Set

Thus, our model for the full data set is

$$\frac{dx}{dt} = 0.005(1-x)(x)^{0.95} - 0.005(1-x)^{0.95}(x).$$

We can now take the solution for the model and plot it with the data, as shown in Figure 8.

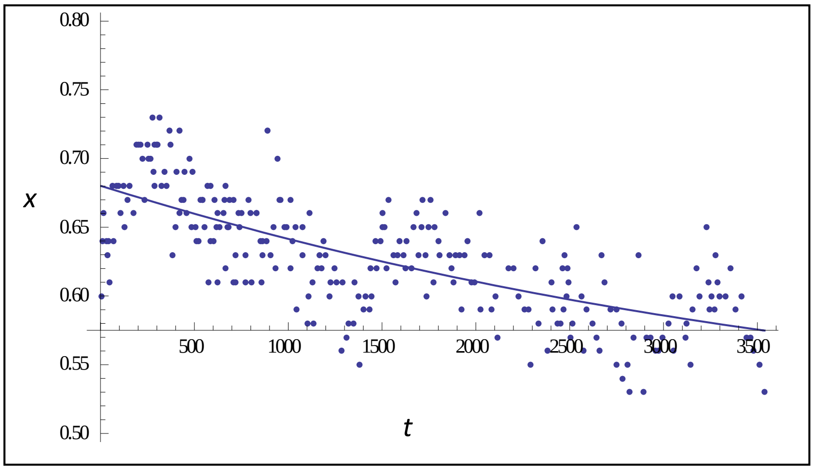

Figure 8: Solution Compared to Data, Full Data Set

Because *a* is less than 1 and u_x is 0.5, we consult Figure 5 to see that x = 1 and x = 0 are unstable equilibrium points. We can use equation (3) to determine that our intermediate equilibrium point is 0.50. This intermediate equilibrium point is stable. Because our initial condition is neither 1 or 0, we would expect this solution to eventually stabilize at x = 0.50. When we project this solution forwards in Figure 9, we see this occurring around t = 25,000.

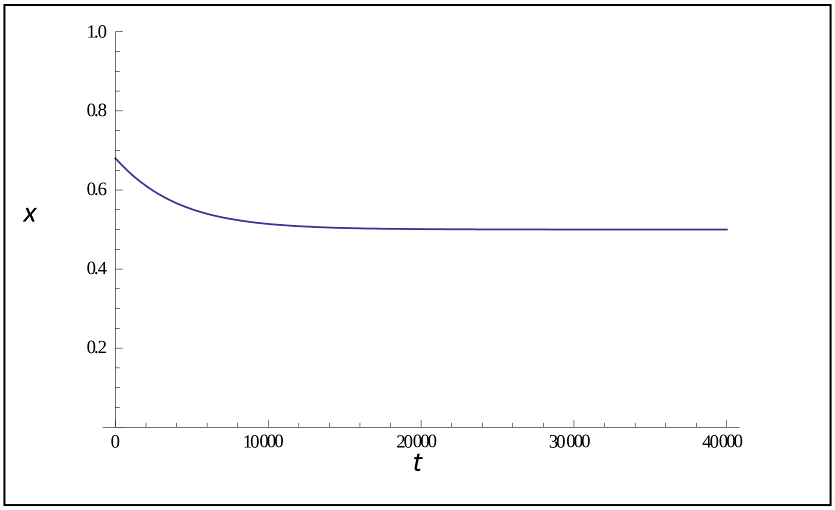

Figure 9: Projected Full Data Set Solution

Since time is measured in days since January 1, 2004, this means that we would expect partisanship to stabilize at 0.50 sometime in 2072.

Now we will consider the data beginning January 1, 2012 and continuing until the November 2012 presidential election. We will now look at this data separately from the full data set in Figure 10.

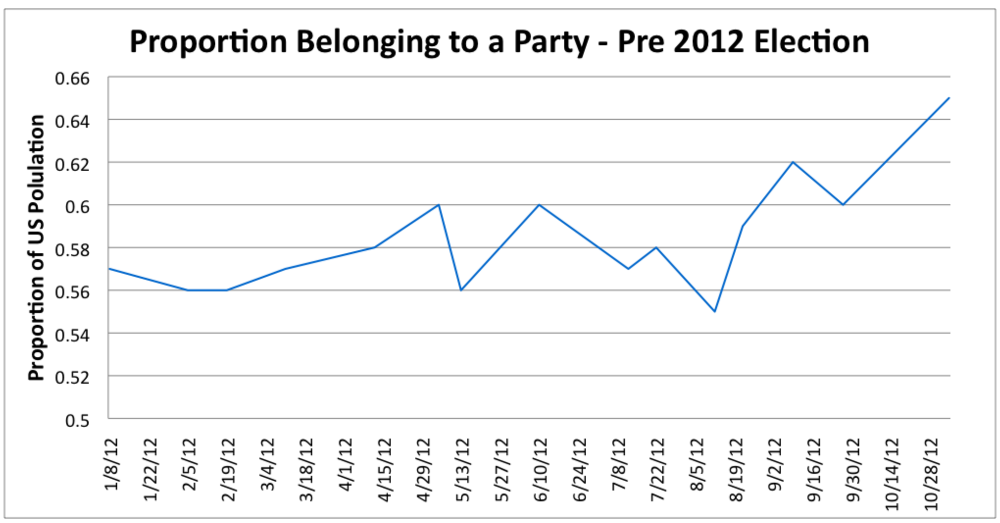

Figure 10: Pre-2012 Election Data

We see in Figure 10, as discussed previously, that this data appears to be trending towards greater partisanship. The parameter ranges and final values are presented in Figure 11.

Parameter	Lower Limit	Upper Limit	Final Value	Step Size
a	1.5	2	1.57	0.01
u_x	0	1	0.46	0.01
c	0	0.25	0.11	0.01
Initial condition	0.55	0.65	0.57	0.01

Figure 11: Parameters for Pre-2012 Election Data

Thus, the model for this data set is

$$\frac{dx}{dt} = 0.0506(1-x)(x)^{1.57} - 0.0594(1-x)^{1.57}(x).$$

The squared error given by these parameter values is approximately 0.003948. We present the solution for this model compared to the data here in Figure 12.

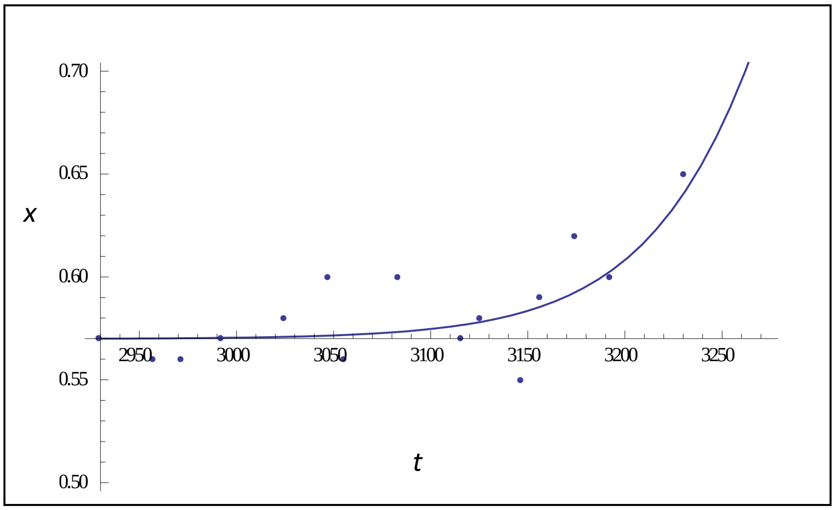

Figure 12: Solution Compared to Data, Pre-2012 Election Data

Because *a* is now greater than 1, while u_x is less than 0.5, Figure 5 tells us that $x = 0$ and $x = 1$ are stable equilibrium points, whereas the intermediate stable point is unstable. The intermediate equilibrium point in this case, as determined by equation (3), is $x \approx 0.5699$. Our initial condition is greater than this intermediate equilibrium point, so we expect our equation to be repelled from this point, increasing towards the stable equilibrium point of x = 1. Projecting this solution forwards, we see in Figure 13 that this occurs around $t = 3{,}500$.

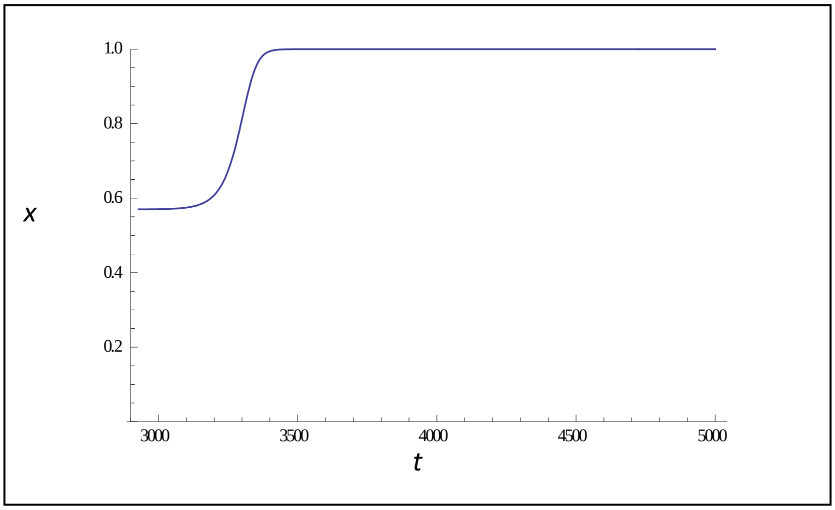

Figure 13: Projected Pre-2012 Election Data Solution

This means that our model suggests that the entire United States population will belong to a party sometime before the end of 2013.

Finally, we will look at data beginning after the November 2012 election and continuing through September 2013. We view this data separately in Figure 14.

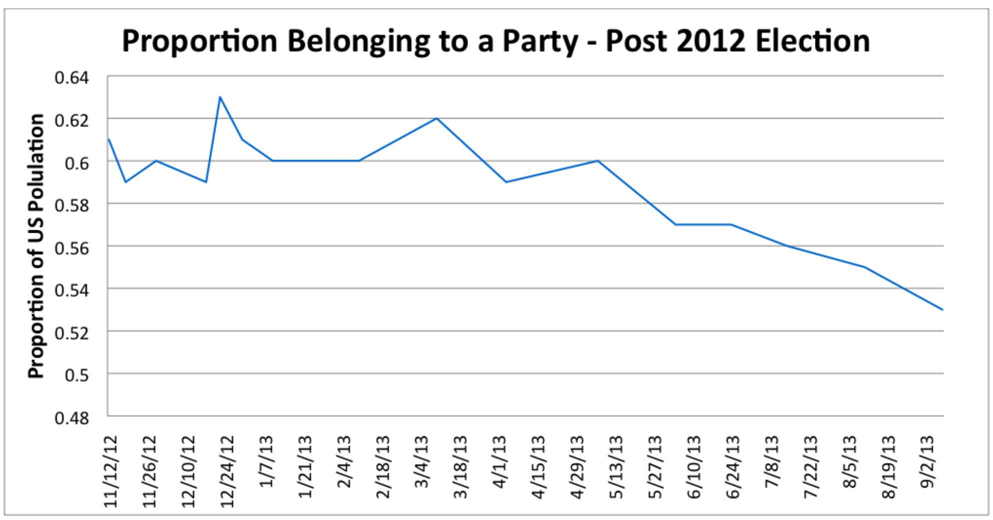

Figure 14: Post-2012 Election Data

We see in Figure 14, as discussed earlier, that data following this election suggests a decline in partisanship. The parameters for the model are shown here in Figure 15.

Parameter	Lower Limit	Upper Limit	Final Value	Step Size
a	1.5	2	1.77	0.01
u_x	0	1	0.41	0.01
c	0	0.25	0.04	0.01
Initial condition	0.53	0.63	0.61	0.01

Figure 15: Parameters for Post-2012 Election Data

The resulting model is

$$\frac{dx}{dt} = 0.0164(1-x)(x)^{1.77} - 0.0236(x)(1-x)^{1.77}.$$

The squared error from this model is approximately 0.0021. We compare the solution to this model to the actual data in Figure 16.

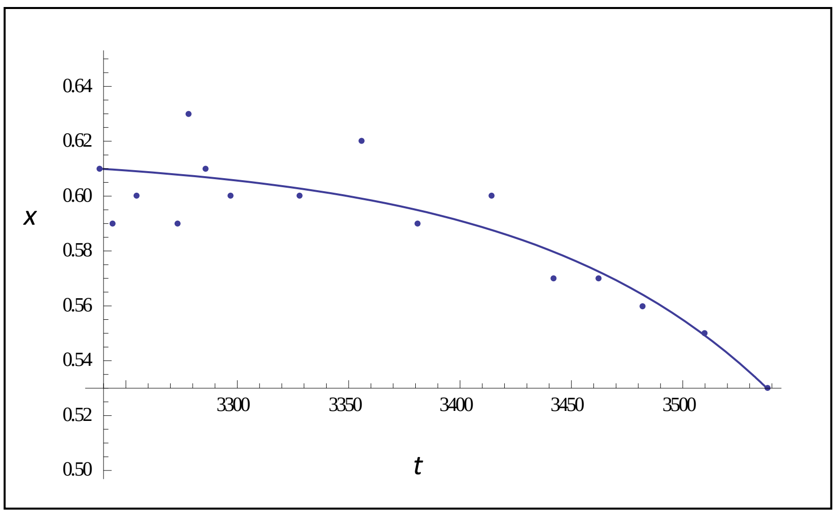

Figure 16: Solution Compared to Data, Post-2012 Election Data

Referring to Figure 5, we see that *a* greater than 1 and u_x less than 0.5 (as in the previous data set) corresponds to *x* = 1 and *x* = 0 being stable equilibrium points. The intermediate equilibrium point as determined by equation (3) is $x \approx 0.6160$. Because our initial condition is now smaller than this intermediate equilibrium point, we expect the solution to be repelled from this point towards the equilibrium point *x* = 0. Projecting the solution forwards in Figure 17, we see that it reaches equilibrium at around *t* = 4,000.

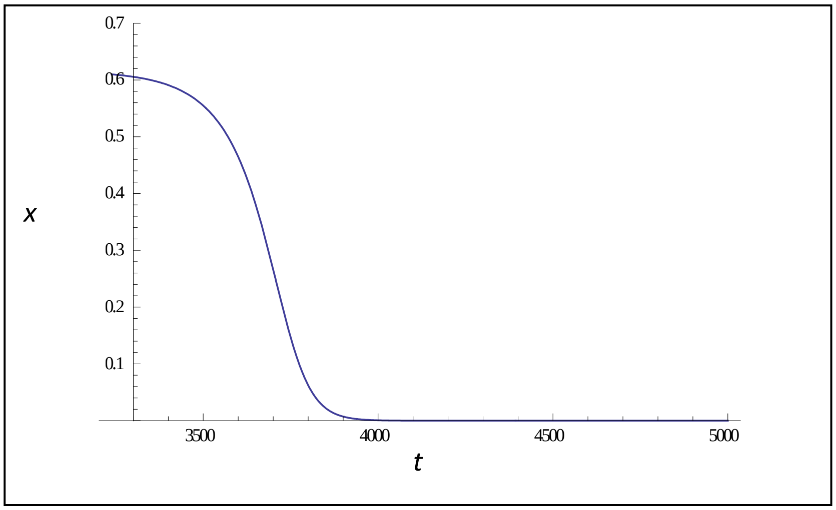

Figure 17: Projected Post-2012 Election Data Solution

This means that this solution projects partisanship to completely disappear by the end of 2014.

4 Conclusion and Directions for Further Research

Based on the results from these three data sets, we can make inferences regarding long- and short-term trends in political partisanship in the United States. Interestingly, with u_x exactly equal to 0.50, our long-term data suggests that the utility of belonging to a party is exactly the same as that of not belonging to a party. However, because *a* in our full data set was slightly less than 1, the majority effect is actually more important than the utility in determining movement. Because partisanship is in the majority for our initial condition of 0.68, we would perhaps expect that partisanship would increase from this point. Instead, it declines towards the equilibrium point of 0.50 – the predicted steady state based on the values for the utility and *a*. This can be explained because *a* is close enough to 1 that it is not enough to overcome the fact that there is more possibility for movement from partisanship to non-partisanship because of the sizes of each group.

From the pre-2012 election model, we see that the utility of belonging to a party is actually smaller than in the long-term model, with u_x equal to 0.46 in this case. This suggests that it was actually more beneficial in the short time prior to the election to *not* belong to a party. With *a* in this case equal to 1.57, we also see that the majority effect is significantly less important than utility regarding movement. The combination of these effects seems to suggest that partisanship should be decreasing due to the higher utility of nonpartisanship. However, we find that partisanship actually increases towards 1. This can be explained because the utility is close enough to 0.50 that it is still not enough of a difference to overcome the majority effect. Thus, with an initial condition of 0.57, partisanship is still in the majority, and attracts members.

Finally, we consider the post-2012 election model. The utility in this case is 0.41 and *a* is 1.77. Thus, it seems as though after the election it became even less valuable to belong to a party than it was before the election. Also, the majority effect has become even less important than it was prior to the election. The combination of these two changes is enough to cause the decrease in partisanship as seen in the solution to the model.

The projection towards 0% or 100% partisanship within the next year or so is, of course, not feasible. These conclusions in our short-term data sets likely occurred because the data does not take the regular fluctuations in political partisanship surrounding election cycles into account. In fact, any result where *a* was greater than or equal to 1 would have led to a similar conclusion, implying that as long as the importance of the majority effect is sufficiently low, we should expect partisanship to either take over completely or to die out. Only when the majority effect has a significant impact (such that *a* is less than 1) does the long-term trend result in a balance between the two. This is what occurred in the model for our full data set.

The fact that we found utility less than or equal to 0.50 in all scenarios is an interesting result. This is perhaps contrary to what we would expect, because the point of belonging to a party is, arguably, because it is beneficial. If, as our long-term model suggests, the utility of belonging to a party is exactly 0.50, then it makes sense that partisanship would eventually settle to 50%. However if, as our short term models suggest, utility is actually less than 0.50, then partisanship is surviving simply due to the majority effect.

There are several strong possibilities for future research surrounding this issue. The simplest way to expand upon this would be to consider different data sets. For example, we could consider the data before and after each election cycle to see if the same pattern holds as that with the 2012 election. We could also attempt to find data that begins further back in time. This way, we could see an even longer-term trend, as well as compare small segments of data to see how utility has changed over time. Another downside to the data used in this study is that all numbers were rounded to only two significant figures. It would improve the accuracy of results if more accurate data could be found.

More research could also be done by expanding upon the current model. Perhaps one of the greatest weaknesses of this model when applied to partisanship data is that the model assumes a static utility. However, in looking at our two short-term data segments, it is easy to see that utility is actually changing over time. If the model could be adapted so that utility was a function of time, a long-term data set could be used to more accurately project future partisanship trends. It could also be interesting to adapt the model so that three groups could be considered simultaneously. In this way, we could consider movement between the two major political parties as well as movement into and away from partisanship.

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