Analysis and Manipulation of the Kruskal Card Trick

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Abstract

In the Kruskal card trick, an audience member has a full suit of cards, ordered ace to King, and the magician has the rest of the deck. The audience member then picks out a card from the magician's deck of his or her choosing, without showing the magician. The audience member has the single suit of cards facing them, ace on top and then silently spells out the name of the chosen card moving a card to the back of the deck with each letter in the name. For instance, if the audience member had chosen a Jack, the audience member would spell out J-A-C-K moving the ace, two, three, and four to the back of the deck. The top card will then be a five. The audience member repeats again this time spelling out F-I-V-E. The audience will repeat the process a third and final time with the next top card and will end up at his or her final card. The magician then makes a guess as to what the final card is, revealing a King to be the final card. The trick will always end up at the King no matter what card the audience member initially spells out when the cards are arranged in order. We move on to three variations of this card trick: how accurate this trick is when the ordered suit is iterated through more than three times, how accurate this trick is when the thirteen cards are shuffled before they are iterated, and how accurate this trick is when both the cards are shuffled, and the number of iterations are changed.

1 Introduction

Magic tricks are simple devices that can amuse any person at first glance. They usually involve some hand motion and distraction of the audience, however there are situations in which the audience is involved in the trick. In a specific card trick, the Kruskal card trick, the audience takes part in counting the names of cards three times and finally landing on a card that the magician will then try to predict. The audience member is shocked to find that the magician guessed the card correctly, but how could the magician possibly know which of the thirteen cards the audience member landed? To the surprise of the audience member, and fellow magic adorers alike, the magician already knew what card the audience member would land. The magician is not doing much guessing at all, but how? Does the magician possess psychic powers, or is there a solution far less surreal, but nonetheless amazing? A magician may not reveal his or her secrets, but a mathematician will.

The trick itself is simple. The audience member has a full suit of cards, ordered ace to King, and the magician has the rest of the deck. The audience member then looks at the remaining cards the magician is holding and picks out a card of his or her choosing, without showing the magician. With that chosen card still in the magician's deck, the audience member has the single suit of cards facing them, ace on top and then silently spells out the name of the chosen

card moving a card to the back of the deck with each letter in the name. For instance, if the audience member had chosen a Jack, the audience member would spell out J-A-C-K moving the ace, two, three, and four to the back of the deck. The top card will then be a five. The audience member repeats again this time spelling out F-I-V-E. The audience will repeat the process a third and final time with the next top card and will end up at his or her final card. At this point, the magician does not have any idea what card the audience member chose, but will make a guess as to what the final card is. The magician reveals a King to be the final card and the unknowing audience member would be in awe to find that this is indeed the final card. Unbeknownst to the crowd, this trick is not magical in nature, but will always end up at the King no matter what card the audience member initially spells out.

The trick works every time, but what happens when you change how many times you spell out the card name? Or what happens when the audience member's deck is shuffled? Or what happens if you do both? This paper explores the aforementioned conditions to determine whether the card trick still yields any success, or if the magician can determine the final card no matter what the situation is.

2 Definitions and Development

To understand the basics of how to perform this trick, or conduct any analysis, we need to know what a deck of cards is. A **card deck** is a collection of 52 unique cards, each with a distinguishing suit and name. There are four suits in a deck of cards, two red suits and two black suits. These suits determine the symbols that adorn the cards. The two red suits are the Hearts and Diamonds and the two black suits are the Spades and Clubs. There are 13 cards of each suit and each of the 13 cards has a different name. The 13 card names, in order of rank, are the Ace, two-ten, the Jack, the Queen, and the King. For the sake of the Kruskal card trick, the Ace card is in place of what could be the number "one" card, and only displays one symbol of whichever suit it belongs. The number cards, two-ten, display the same number of symbols for which the card is named. For example, the Two of Hearts would have two red hearts on it. The later mentioned names, the Jack, Queen, and King, are referred to as the face cards. They are nicknamed the face cards because instead of having symbols, they have human-like figures, namely a Jack, a Queen, and a King, decorated in their representative suit. For the trick, the audience member will have one full suit with the cards ordered from Ace, two-ten, Jack, Queen, and King.

Definition 1 A **cyclic permutation** is a permutation that shifts all elements of a set by a fixed number and moves the elements shifted off the edge to the back of the set. The number of cyclic permutations is denoted by $k \in \mathbb{Z}_+$.

In the case of the Kruskal card trick, the fixed number is number of letters it takes to spell out the name of a card. The fixed number changes for each cyclic permutation performed. The audience member will then perform an indicated number of cyclic permutations, k, on the cards to achieve what we will call the **final card**, the card on top of the deck after the indicated number of cyclic permutations. In the standard version of the trick, we have $k = 3$.

Example 2 Let $k = 2$, and the audience members chosen card to be a Queen. The cyclic permutations are as follows,

$$k = 0, \quad \{A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K\},$$

$$k = 1, \quad \{6, 7, 8, 9, 10, J, Q, K, A, 2, 3, 4, 5\},$$

$$k = 2, \quad \{9, 10, J, Q, K, A, 2, 3, 4, 5, 6, 7, 8\}.$$

After two permutations, the final card is a 9. Notice after the $k = 1$ case, the fixed number changed from five, indicated by the number of letters in Queen, to three because the name of the top card, 6, has three letters.

3 Results

The introduction mentioned three conditions that this paper will be exploring. The first variation of the card trick is simple—what happens if we change the number of cyclic permutations the card trick is performed requested by the magician? If we increase the number of cyclic permutations, we analyze cyclic properties appearing, as indicated in Table 1. Since after three cyclic permutations every starting card ends on the King, we can generalize the results for only one case instead of three. We find that the final cards eventually become cyclic in nature and can be determined with modular arithmetic, $k \bmod 3$. The group $\mathbb{Z}_3 = \{0, 1, 2\}$ corresponds to the set of cards $\{K, 4, 8\}$. As long as the magician remembers the corresponding cards with the number of cyclic permutations, the magician will always be able to make a correct prediction.

k	Final Card	k	Final Card
4	4	9	K
5	8	10	4
6	K	11	8
7	4	12	K
8	8	13	4

Table 1: Cyclic Permutations

The second variation of the card trick becomes more complicated. In this second variation, the 13 cards are shuffled before the cyclic permutations are performed. In this version of the trick, it is not the case that the magician can just know what the final card is based on the position in the deck, but the magician needs the trick to end on the 13th card in the shuffled deck, much like the standard trick. The way the magician finds out the 13th card all depends on how the cards are arranged. Instead of shuffling the already separated suit, the magician shuffles the full deck of 52 cards. The bottom card of the shuffled deck is then visible for the magician to be seen. The magician then hands this entire shuffled deck over to the audience member and asks the audience member to sort out a suit using that visible bottom card. Hopefully, if the audience member sorts the cards in a typical manner, that visible card would be the first one set down and would turn into the card in the 13th spot. Then, the rest of the cards set down to obtain the suit will also be shuffled.

Analyzing this version of the trick required the work of a computer program to simulate the card trick. The program shuffled 13 cards, determined the final card, used a random number generator to pick a starting card, then performed the cyclic permutations three times. This simulation was then run 1,000,000 times in order to calculate the probability that the final card

will be the 13th card in the suit deck. The program determined that the trick drops to a 28.6% accuracy. This means that only 28.6% of the time, when the suit is shuffled, will the final card be the same as the 13th card in the suit. This is a significant drop from the 100% accuracy of the standard trick.

The third and final variation of the trick combines both the first and second variations. We analyze what happens if we have the shuffled suit and change the number of cyclic permutations. To do this, we use the same computer program and plug in different values for the number of cyclic permutations. Figure 1 shows the relationship between the number of cyclic permutations and the percentage of the time the trick ends in the 13th card when the 13 cards are shuffled. As we can see, when we have k values that are multiples of 3, the number of permutations that the original trick has, the percentage of accuracy is much higher than the in between numbers. However, the trick never reaches above 28.6% accuracy for any value of k.
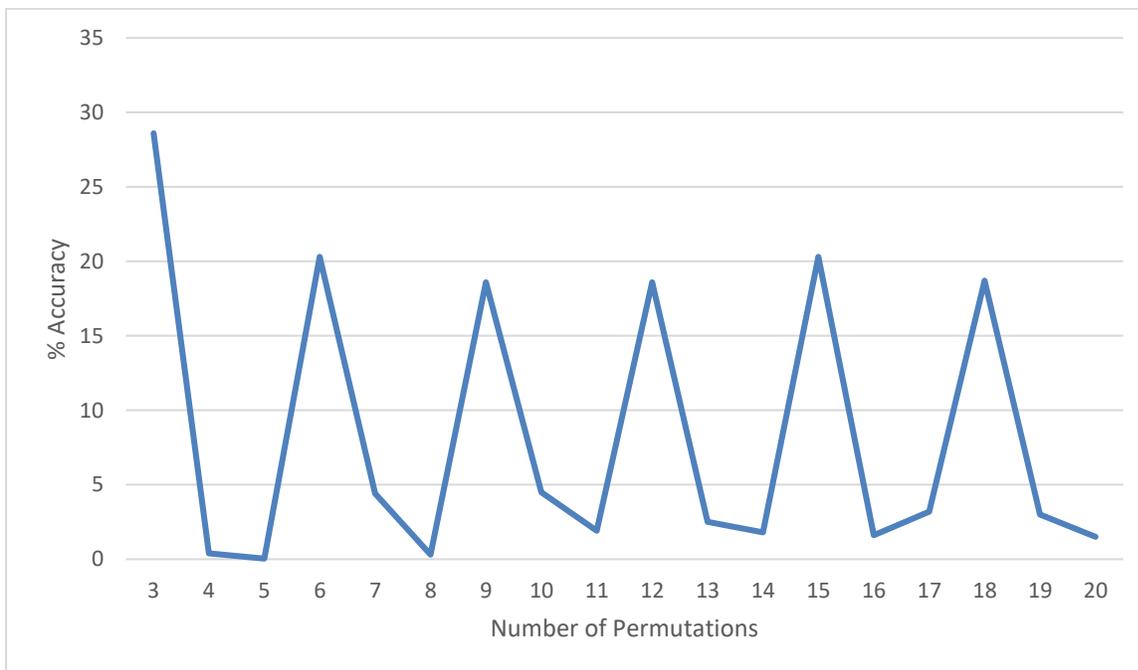

Figure 1: Accuracy of Permutations

4 Conclusion and Directions for Further Research

Though the different variations of the trick offer different changes in the accuracy and outcome, there is nothing that beats the standard trick. Variation 1 maintains its accuracy, meaning the magician will always be able to determine the final card the audience member holds. However, like the standard trick, this version loses its magical qualities as over time it would be deemed unimpressive as the audience will eventually catch onto the cyclic nature of the cards or that the magician continuously guesses the same cards. On the other hand, variation 2 has a significant drop in the accuracy of the trick. With over 70% loss in the certainty of the outcome, the favor of the trick leans towards the audience member. This variation is extremely risky to the magician since majority of the time, he or she will have no idea what the final card turns out to be. On the

off chance, that the magician does end up winning out over the audience member, the trick brings on a magical quality that is lacking in the previous variation and the standard trick. Shuffling the cards would give the magician 13 cards to choose from making it more impressive to the audience member if guessed correctly. Variation 3 combines both previous variations and leads to more ineffective results. The accuracy drops to nearly 0% in certain iteration numbers, and though it would be impressive if guessed correctly, the final card would be extremely hard to accurately obtain when many iteration cycles do not end near the 13th card. This variation of the card trick would be extremely unlikely for the magician to perform, as he or she would lose majority of the time.

Though there are many options this card trick could take for other variations, the most interesting direction would be to observe what happens in any subset of 13 cards. Instead of having one of each card in a suit, if we could have more than one of any face or numeric card, what would happen? The standard trick has 100% accuracy and the shuffled version has 28.6% accuracy. Is it possible to find another subset of cards that has an accuracy between that of the standard version and the shuffled version?

References

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