Using Markov Chains to Analyze Volleyball Matches

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Abstract

In this study we examine how to model a volleyball match with the use of Markov chains and a computer program. We then use this model to examine the importance of different aspects of the game.

1 Introduction

Volleyball is a highly competitive game for athletes and coaches alike. Each practice, coaches must focus on what area the team needs to improve to prepare for the next game and the players must also share this vision. Using precise statistics from recorded matches of the Carthage College women's volleyball team, we were able to generate probabilities that a ball would transition between the various states in a volleyball game. We then used Markov Chains to analyze theses probabilities by entering them into transition matrices to determine the probability of a team scoring a point based on which team served. Then entering these probabilities into a computer program and simulating matches, we determined which team overall performed better. Then we were able to adjust the original statistics to see if small improvements to a specific area would have changed the outcomes of the matches.

2 Definitions and Development

For our research, we recorded two volleyball matches. Then we analyzed the videos to generate the statistics in each match. These statistics were then used to derive the probabilities of the ball entering into the various states.

Definition 1 A **serve** is a hit from one team to the other used to put the ball in play [2].

Definition 2 A **pass** is contact with the ball with the intent to control the ball to another player [2].

Since there can be at most three touches each time the ball is on one side of the court, there can be at most two passes. Therefore we designate two passing states, Pass1 for the first pass of the ball and Pass2 for the second pass.

Definition 3 A **set** is a skill in which a ball is directed to a point where a player can hit it into the opponent's court [2].

Definition 4 An **attack**, also referred to as a hit, kill, or spike, occurs when a ball is contacted with force by a player on the offensive team who intends to terminate the ball on the opposing team's court or off the opponent's block [2].

Definition 5 A **free ball** is a return of the ball to the opponent without the intent to get a point [2].

Definition 6 A **block** is a defensive play by one or more players meant to intercept a spiked ball [2].

Definition 7 A **point** occurs when the ball hits the court or a player makes an illegal move [2].

Definition 8 A **state** in volleyball is the position the ball is in during a rally. For the purposes of this paper, the states are Serve, Pass1, Pass2, Set, Hit, Free Ball, Block, and Point.

Definition 9 A **Markov Chain** is usually a discrete stochastic process in which the probabilities of occurrence of various future states depend only on the present state of the system or on the immediately preceding state, not on the path by which the present state was achieved [1].

Definition 10 A state is considered to be **absorbing** if it is impossible to leave that state. In a volleyball match only Point states are absorbing [1].

Definition 11 A **transition matrix** is a collection of transition probabilities arranged in a square matrix [1].

To distinguish a state between Carthage and the opponent, each state was labeled as either 1 or 2. States which are related to Carthage are followed by the label 1 and states which are related to the opponent are followed by the label 2. Figure 1 demonstrates how the matrices were composed. The first match was Carthage against University of Wisconsin Eau Claire, so the state Hit 1 represents a Carthage hit and Point 2 indicates a UW-Eau Claire point.

Definition 12 A **transition probability** is the calculated probability of moving from one state to the next.

Definition 13 A **scoring probability** is the calculated probability of one team scoring after any given rally. It is found by raising the transition matrix to an arbitrarily large number, then multiplying by the team's serving vector.

We then entered the scoring probabilities into a computer program (see appendix) which ran five trials of 100 simulated games.

3 Results

We can now begin to analyze whether the created model accurately represents volleyball matches. Two volleyball matches were recorded to obtain the transition probabilities for this

	Serve 1	*Pass* 1 1	*Pass*2 1	*Set* 1	*Hit* 1	*Free Ball*1	*Block* 1	*Serve* 2	*Pass*1 2	*Pass*2 2	*Set* 2	*Hit* 2	*Free Ball* 2	*Block* 2	*Point* 1	*Point* 2
Serve 1	0	0	0	0	0	0	0	0	0.83	0	0	0	0	0	0.06	0.11
*Pass*1 1	0	0	0.23	0.65	0.06	0.03	0	0	0	0	0	0	0	0	0	0.03
*Pass*2 1	0	0	0	0	0.49	0.37	0	0	0	0	0	0	0	0	0	0.14
Set 1	0	0	0	0	0.98	0.01	0	0	0	0	0	0	0	0	0	0.01
Hit 1	0	0	0	0	0	0	0	0	0.36	0	0	0	0.01	0.23	0.26	0.14
Free Ball 1	0	0	0	0	0	0	0	0	0.95	0	0	0	0	0	0	0.05
Block 1	0	0.24	0	0	0	0	0	0	0.36	0	0	0.02	0.05	0	0.07	0.26
Serve 2	0	0.88	0	0	0	0	0	0	0	0	0	0	0	0	0.09	0.03
*Pass*1 2	0	0	0	0	0	0	0	0	0	0.31	0.59	0.05	0.03	0	0.02	0
*Pass*2 2	0	0	0	0	0	0	0	0	0	0	0	0.66	0.25	0	0.09	0
Set 2	0	0	0	0	0	0	0	0	0	0	0	0.95	0.03	0	0.02	0
Hit 2	0	0.45	0	0	0	0	0.36	0	0	0	0	0	0	0	0.02	0.17
Free Ball 2	0	0.7	0	0	0.15	0	0	0	0	0	0	0	0	0	0.1	0.05
Block 2	0	0.32	0	0	0	0.04	0	0	0.32	0	0	0	0	0	0.21	0.11
Point 1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
Point 2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1

Figure 1: Match 1 Transition Matrix.

analysis. Figure 1 shows the transition matrix from match one, Carthage vs. UW-Eau Claire. For example, the probability of a Carthage Hit resulting in a UW-Eau Claire Block is 0.23.

We then raised match 1's transition matrix by a power of 100 and multiplied by the vector (1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0), Carthage's serving vector, in order to determine the scoring probability for Carthage. To determine UW-Eau Claire's scoring probability, we multiplied by the vector (0,0,0,0,0,0,0,1,0,0,0,0,0,0,0,0). Any arbitrarily large number yields the same results since it is unlikely that the ball would still be in play after 100 touches. We found that if Carthage served the ball they would score with a probability of 0.42. If UW-Eau Claire served they would score with a probability of 0.48.

We developed the computer program to simulate volleyball games using the scoring probabilities of each team. The program started each game with Carthage serving, so Carthage's scoring probability was used to determine which team scored the first point. Then the winning team's scoring probability was used for the next rally. This process repeated until one team reached 25 points. Then the program reset the score and repeated the process until 1000 games were simulated. The program then repeated the whole process four more times then outputted the results of the five trials.

After running the scoring probabilities of Carthage and UW-Eau Claire, we found that UW-Eau Claire won an average of 661 out of 1000 games. In the original match, UW-Eau Claire won 3-0.

The second analysis is of the match Carthage vs. UW-Platteville. The transition matrix of this match can be seen in Figure 2.

	Serve 1	*Pass* 1 1	*Pass*2 1	*Set* 1	*Hit* 1	*Free Ball*1	*Block* 1	*Serve* 2	*Pass*1 2	*Pass*2 2	*Set* 2	*Hit* 2	*Free Ball* 2	*Block* 2	*Point* 1	*Point* 2
Serve 1	0	0	0	0	0	0	0	0	0.87	0	0	0	0.02	0	0.06	0.05
*Pass*1 1	0	0	0.21	0.66	0.07	0.02	0	0	0	0	0	0	0	0	0	0.04
*Pass*2 1	0	0	0	0	0.7	0.16	0	0	0	0	0	0	0	0	0	0.14
Set 1	0	0	0	0	0.95	0.01	0	0	0	0	0	0	0	0	0	0.04
Hit 1	0	0	0	0	0	0	0	0	0.42	0	0	0	0.01	0.25	0.26	0.06
Free Ball 1	0	0	0	0	0	0	0	0	0.53	0	0	0.28	0	0	0	0.18
Block 1	0	0.33	0	0	0	0	0	0	0.2	0	0	0.03	0.05	0	0.21	0.18
Serve 2	0	0.86	0	0	0	0.03	0	0	0	0	0	0	0	0	0.1	0.01
*Pass*1 2	0	0	0	0	0	0	0	0	0	0.28	0.65	0.04	0.02	0	0.01	0
*Pass*2 2	0	0	0	0	0	0	0	0	0	0	0	0.73	0.23	0	0.04	0
Set 2	0	0	0	0	0	0	0	0	0	0	0	0.96	0.02	0	0.02	0
Hit 2	0	0.43	0	0	0	0.02	0.25	0	0	0	0	0	0	0	0.09	0.21
Free Ball 2	0	0.61	0	0	0.17	0	0	0	0	0	0	0	0	0	0.22	0
Block 2	0	0.18	0	0	0.03	0.03	0	0	0.26	0	0	0.03	0.03	0	0.26	0.18
Point 1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
Point 2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1

Figure 2: Match 2 Transition Matrix.

Again we raised the transition matrix by a power of 100, and then multiplied this product by first Carthage's serving vector, then UW-Platteville's serving vector. We found in the second match when Carthage served they would score with a probability of 0.51. When UW-Platteville served they would score with a probability of 0.42.

We entered these scoring probabilities into the computer program and discovered Carthage won an average of 740 games. In the original match, Carthage beat UW-Platteville 3-1.

The next area of our research looks at modifying specific areas of Carthage's play to determine if the outcomes could have been different had Carthage played better. To do this we increased or decreased a transition probability by 10%. The first adjustment we looked at was increasing Carthage's killing percentage. For this increase we considered if the hits that landed out of play would have landed in the UW-Eau Claire court. For this adjustment we increased the probability a Carthage Hit would result in a Carthage Point from 0.26 to 0.29. This the led to a decrease of the probability a Carthage Hit resulted in a UW-Eau Claire Point from 0.14 to 0.11. See Figure 3.

This new transition matrix was raised by a power of 100 and then multiplied by each team's serving vector to determine each team's scoring probability. This adjustment increased Carthage's scoring probability from 0.42 to 0.44 and decreased UW-Eau Claire's scoring probability from 0.48 to 0.45. After entering the scoring probabilities into the computer program, we discovered that UW-Eau Claire won an average of 540 games, a much closer outcome than in the original match.

	Serve 1	*Pass* 1 1	*Pass*2 1	*Set* 1	*Hit* 1	*Free Ball*1	*Block* 1	*Serve* 2	*Pass*1 2	*Pass*2 2	*Set* 2	*Hit* 2	*Free Ball* 2	*Block* 2	*Point* 1	*Point* 2
Serve 1	0	0	0	0	0	0	0	0	0.83	0	0	0	0	0	0.06	0.11
*Pass*1 1	0	0	0.23	0.65	0.06	0.03	0	0	0	0	0	0	0	0	0	0.03
*Pass*2 1	0	0	0	0	0.49	0.37	0	0	0	0	0	0	0	0	0	0.14
Set 1	0	0	0	0	0.98	0.01	0	0	0	0	0	0	0	0	0	0.01
Hit 1	0	0	0	0	0	0	0	0	0.36	0	0	0	0.01	0.23	0.29	0.11
Free Ball 1	0	0	0	0	0	0	0	0	0.95	0	0	0	0	0	0	0.05
Block 1	0	0.24	0	0	0	0	0	0	0.36	0	0	0.02	0.05	0	0.07	0.26
Serve 2	0	0.88	0	0	0	0	0	0	0	0	0	0	0	0	0.09	0.03
*Pass*1 2	0	0	0	0	0	0	0	0	0	0.31	0.59	0.05	0.03	0	0.02	0
*Pass*2 2	0	0	0	0	0	0	0	0	0	0	0	0.66	0.25	0	0.09	0
Set 2	0	0	0	0	0	0	0	0	0	0	0	0.95	0.03	0	0.02	0
Hit 2	0	0.45	0	0	0	0	0.36	0	0	0	0	0	0	0	0.02	0.17
Free Ball 2	0	0.7	0	0	0.15	0	0	0	0	0	0	0	0	0	0.1	0.05
Block 2	0	0.32	0	0	0	0.04	0	0	0.32	0	0	0	0	0	0.21	0.11
Point 1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
Point 2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1

Figure 3: Match 1 Increased Killing Percentage Transition Matrix.

The second adjustment we looked at was increasing Carthage's passing success. For this increase we considered if the first pass from Carthage would have resulted in a set instead of a point for the opponent. This increased the probability the first pass resulted in a set from 0.65 to 0.68. It should be noted that this is only an increase of 4.6%, this is due to the fact the probability of Carthage's first pass resulting in a UW-Eau Claire point was only 0.03 in the original transition matrix, so could only be decreased by 3 percentage points. See Figure 4.

	Serve 1	*Pass* 1 1	*Pass*2 1	*Set* 1	*Hit* 1	*Free Ball*1	*Block* 1	*Serve* 2	*Pass*1 2	*Pass*2 2	*Set* 2	*Hit* 2	*Free Ball* 2	*Block* 2	*Point* 1	*Point* 2
Serve 1	0	0	0	0	0	0	0	0	0.83	0	0	0	0	0	0.06	0.11
*Pass*1 1	0	0	0.23	0.68	0.06	0.03	0	0	0	0	0	0	0	0	0	0
*Pass*2 1	0	0	0	0	0.49	0.37	0	0	0	0	0	0	0	0	0	0.14
Set 1	0	0	0	0	0.98	0.01	0	0	0	0	0	0	0	0	0	0.01
Hit 1	0	0	0	0	0	0	0	0	0.36	0	0	0	0.01	0.23	0.26	0.14
Free Ball 1	0	0	0	0	0	0	0	0	0.95	0	0	0	0	0	0	0.05
Block 1	0	0.24	0	0	0	0	0	0	0.36	0	0	0.02	0.05	0	0.07	0.26
Serve 2	0	0.88	0	0	0	0	0	0	0	0	0	0	0	0	0.09	0.03
*Pass*1 2	0	0	0	0	0	0	0	0	0	0.31	0.59	0.05	0.03	0	0.02	0
*Pass*2 2	0	0	0	0	0	0	0	0	0	0	0	0.66	0.25	0	0.09	0
Set 2	0	0	0	0	0	0	0	0	0	0	0	0.95	0.03	0	0.02	0
Hit 2	0	0.45	0	0	0	0	0.36	0	0	0	0	0	0	0	0.02	0.17
Free Ball 2	0	0.7	0	0	0.15	0	0	0	0	0	0	0	0	0	0.1	0.05
Block 2	0	0.32	0	0	0	0.04	0	0	0.32	0	0	0	0	0	0.21	0.11
Point 1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
Point 2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1

Figure 4: Match 1 Increased Passing Success Transition Matrix.

We then raised this matrix by the power of 100 and multiplied by each team's serving matrix to determine each team's scoring probabilities. This adjustment increased Carthage's scoring probability from 0.42 to 0.43 and decreased UW-Eau Claire's scoring probability from 0.48 to 0.46. We entered these scoring probabilities into the computer program and discovered UW-Eau Claire won an average of 586 games. This was less than in the original match, but not as great of a difference as the first adjustment.

Next we made the same adjustments to the probabilities in the second match. Note that since Carthage won the second match we were not looking for a change in outcome, but rather if the adjustments made to the second match's transition matrix were consistent with the adjustments to the first match. The first adjustment we made to the second match was increasing Carthage's killing percentage. For this increase we again focused on considering if the hits that landed out of play would have landed in the UW-Platteville court. For this adjustment we increased the probability a Carthage Hit would result in a Carthage Point from 0.26 to 0.29. See Figure 5. This then led to a decrease of the probability a Carthage Hit resulted in a UW-Platteville Point from 0.06 to 0.03.

We raised this new transition matrix by a power of 100 and then multiplied by each team's serving vector to determine each team's scoring probability. This adjustment increased Carthage's scoring probability from 0.51 to 0.53 and decreased UW-Platteville's scoring probability from 0.42 to 0.39. After entering the scoring probabilities into the computer program we discovered that Carthage won an average of 838 games, 98 more games than the original match.

	Serve 1	Pass 1 1	Pass2 1	Set 1	Hit 1	Free Ball1	Block 1	Serve 2	Pass1 2	Pass2 2	Set 2	Hit 2	Free Ball 2	Block 2	Point 1	Point 2
Serve 1	0	0	0	0	0	0	0	0	0.87	0	0	0	0.02	0	0.06	0.05
Pass1 1	0	0	0.21	0.66	0.07	0.02	0	0	0	0	0	0	0	0	0	0.04
Pass2 1	0	0	0	0	0.7	0.16	0	0	0	0	0	0	0	0	0	0.14
Set 1	0	0	0	0	0.95	0.01	0	0	0	0	0	0	0	0	0	0.04
Hit 1	0	0	0	0	0	0	0	0	0.42	0	0	0	0.01	0.25	0.29	0.03
Free Ball 1	0	0	0	0	0	0	0	0	0.53	0	0	0.28	0	0	0	0.18
Block 1	0	0.33	0	0	0	0	0	0	0.2	0	0	0.03	0.05	0	0.21	0.18
Serve 2	0	0.86	0	0	0	0.03	0	0	0	0	0	0	0	0	0.1	0.01
Pass1 2	0	0	0	0	0	0	0	0	0	0.28	0.65	0.04	0.02	0	0.01	0
Pass2 2	0	0	0	0	0	0	0	0	0	0	0	0.73	0.23	0	0.04	0
Set 2	0	0	0	0	0	0	0	0	0	0	0	0.96	0.02	0	0.02	0
Hit 2	0	0.43	0	0	0	0.02	0.25	0	0	0	0	0	0	0	0.09	0.21
Free Ball 2	0	0.61	0	0	0.17	0	0	0	0	0	0	0	0	0	0.22	0
Block 2	0	0.18	0	0	0.03	0.03	0	0	0.26	0	0	0.03	0.03	0	0.26	0.18
Point 1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
Point 2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1

Figure 5: Match 2 Increased Killing Percentage Transition Matrix.

Next we adjusted Carthage's passing success. Again we considered the scenario if more of Carthage's first passes had resulted in sets rather than opponent points. For this adjustment we modified the probability a Carthage Pass1 resulted in a Carthage Set from 0.66 to 0.7 and also decreased the probability a Carthage Pass1 resulted in a UW-Platteville Point from 0.04 to 0. It should be noted that only a 6% increase was made because the probability a Carthage Pass1 resulted in a UW-Platteville Point could only be reduced by four percentage points. The modified transition matrix can be seen in Figure 6.

	Serve 1	Pass 1 1	Pass2 1	Set 1	Hit 1	Free Ball1	Block 1	Serve 2	Pass1 2	Pass2 2	Set 2	Hit 2	Free Ball 2	Block 2	Point 1	Point 2
Serve 1	0	0	0	0	0	0	0	0	0.87	0	0	0	0.02	0	0.06	0.05
Pass1 1	0	0	0.21	0.66	0.07	0.02	0	0	0	0	0	0	0	0	0	0.04
Pass2 1	0	0	0	0	0.7	0.16	0	0	0	0	0	0	0	0	0	0.14
Set 1	0	0	0	0	0.95	0.01	0	0	0	0	0	0	0	0	0	0.04
Hit 1	0	0	0	0	0	0	0	0	0.42	0	0	0	0.01	0.25	0.26	0.06
Free Ball 1	0	0	0	0	0	0	0	0	0.53	0	0	0.28	0	0	0	0.18
Block 1	0	0.33	0	0	0	0	0	0	0.2	0	0	0.03	0.05	0	0.21	0.18
Serve 2	0	0.86	0	0	0	0.03	0	0	0	0	0	0	0	0	0.1	0.01
Pass1 2	0	0	0	0	0	0	0	0	0	0.28	0.65	0.04	0.02	0	0.01	0
Pass2 2	0	0	0	0	0	0	0	0	0	0	0	0.73	0.23	0	0.04	0
Set 2	0	0	0	0	0	0	0	0	0	0	0	0.96	0.02	0	0.02	0
Hit 2	0	0.43	0	0	0	0.02	0.25	0	0	0	0	0	0	0	0.09	0.21
Free Ball 2	0	0.61	0	0	0.17	0	0	0	0	0	0	0	0	0	0.22	0
Block 2	0	0.18	0	0	0.03	0.03	0	0	0.26	0	0	0.03	0.03	0	0.26	0.18
Point 1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
Point 2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1

Figure 6: Match 2 Increased Passing Success Transition Matrix.

This modified transition matrix was then raised to the power 100 and multiplied by each team's serving vectors in order to determine the new scoring probabilities. Carthage's scoring probability increased from 0.51 to 0.53 and UW-Platteville's scoring probability decreased from 0.42 to 0.39. These new scoring probabilities were then inputted into the computer program. After running the program we learned that Carthage again won an average of 98 more games than in the original match.

4 Conclusion and Directions for Further Research

The combination of the Markov Chains and computer program was able to determine which team won and lost in each match in five independent trials. Based on the analyses of match 1 and match 2, the system we designed was able to recreate the original outcomes, therefore we can conclude that this is an accurate model for volleyball matches. Additionally, the modifications we made to each match yielded consistent results. It was shown in two independent matches that increasing a team's killing percentage and passing success led to an increase in games won in the computer simulation. Therefore we can also conclude that effective hitting and passing are key areas of a volleyball match. However, based on the results we cannot determine which is more effective or if these two areas are the most important areas of the game. Future research could be done to find the most influential aspect of a volleyball match. More data would need to be found and analyzed statistically in order to prove any assertions made.

The results of this research could be applied by coaches to improve team's performance. Rather than use actual statistics from matches, coaches could use complied statistics throughout the season of their team and opponents to find transition probabilities. Then the coach could use the methods discussed to see if one area, which can be easily practiced and improved, can lead to a better predicted outcome.

References

[1] Best, S., Using Markov Chains to Analyze a Volleyball Rally, http://dspace.carthage.edu/xmlui/bitstream/handle/123456789/406/SBest_Thesis3.pdf?sequence=1, (2010).

[2] Glossary, Federation Internationale de Volleyball, www.fivb.org/en/volleyball/Glossary.asp, (2011).

Appendix

Computer Program

```
#include <iostream>
#include <cstdlib>
#include <ctime>
#include <string>
using namespace std;

int main()
{       srand(time(NULL));
        double carserve;
        double oppserve;

        while(true)
        {

                cout << "Enter Carthage's serve win probability: " << endl;
                cin >> carserve;
                cout << "Enter opponent's serve win probability: " << endl;
                cin >> oppserve;
            for( int j=1; j<=5; j++)
            {

                int cargame = 0;
                int oppgame = 0;

                        for( int i=1; i<=1000; i++)
                        {
                            int carpoint = 0;
                            int opppoint = 0;
                            int server = 0;

                            while((carpoint < 25) && (opppoint < 25))
                            {

                                if((server % 2) == 0)
                                {
                                    if(rand() % 101 / 100.0 <= carserve)
                                    {
                                        carpoint ++;
                                    }
                                    else
                                    {
                                        opppoint ++;
                                        server ++;
                                    }
                                }
```

```
                    else
                    {
                        if(rand() % 101/100.0 <= oppserve)
                        {
                            opppoint ++;
                         }
                        else
                        {
                            carpoint ++;
                            server ++;
                         }
                    }
                }
                if( carpoint > opppoint)
                {
                    cargame++;
                }
                else
                {
                    oppgame++;
                }
            }


            cout << "The result of the match is Carthage " << cargame << endl;
            cout << "and Opponent " << oppgame << "." << endl;


        }
    string choice;
    cout << "Want to end?" << endl;
    cin >> choice;
    if(choice != "n")
        break;
    }


    system("pause");
    return 0;
}
```