Surviving the End of the World: A Mathematical Model of the Zombie Apocalypse

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Abstract

1 Introduction

The idea of the undead returning to plague the living is prevalent throughout history and in a wide variety of cultures. The earliest recorded occurence of zombies in mythology seems to be in the Epic of Gilgamesh - one of the oldest written stories in existence - when the goddess Ishtar declares that she will "let the dead to go up and eat the living."[2] Norse mythology include *draugar*, animated corpses living in the tombs where they were laid to rest. Chinese lore has the Jiang Shi, a corpse that rises at night to prey on the living. Perhaps the most widely recognized zombie is the Voodoo zombie, a person brought back from the dead by a sorcerer or using magic. The zombie as it is known in western culture today, however, has its origin in George A. Romero's film *Night of the Living Dead*. Released in 1968, the film introduced zombies as they are thought of now - mindless, insatiable monsters that strike in hordes.[3] Since the release of Romero's cult classic, zombies have experienced a rapidly expanding popularity in pop culture. From books like *World War Z* to television shows such as *The Walking Dead* and video games - Call of Duty has a zombie fighting game that gives players the chance to fight a zombie Romero - zombies are among us.

For many, the rise in exposure to zombies has encouraged the development of a zombie apocalypse survival plan. These can be as simple as keeping a disaster emergency kit nearby or as complex as buying a fortified house. However, because there is such a wide variety of survival strategies, many are bound to fail. This paper compares different zombie outbreaks and human reactions to determine the best course of action for humanity's continued existence.

2 Definitions and Development

The abilities of undead creatures vary wildly across different mythologies, so we choose the slow-moving zombie from pop culture, best explained in *The Zombie Survival Guide*, as our standard zombie.[1] Because of their speed, we assume that zombies will generally be less efficient at infecting people than people are at killing them. This assumption leads to the formation of our basic model. We begin by dividing the population into three classes: S, the survivors; Z, the zombies; and D, the deceased. The deceased category consists of humans who have died from natural causes and zombies who have been defeated by humans. We use β to represent the rate of zombism transmission, α to represent the rate at which humans destroy zombies, and π and δ as the human birth and death rates, respectively. Additionally, μ is the rate at which humans kill other humans, intentionally or accidentally; μ is a function of time, defined as $\frac{0.015}{t}$. This function was chosen because as humans acclimate to a post-apocalypse Earth, the rate at which incidents like zombie misidentification and fights for supplies occur will most likely shrink.

Now that we have coefficients determined, applying them to interactions between classes will yield our model. There are two interactions to consider - interactions between two survivors and interactions between a survivor and a zombie. The chance that any survivor will interact with any zombie is SZ, so the number of people killed by zombies is βSZ. Similarly, the chance of two survivors interacting is S^2; this means the number of survivors killed by another survivor is μS^2. The number of people born and dying of natural causes can be represented as πS and δS respectively. Combining these terms yields the equation for survivors, $\dot{S} = \pi S - \beta SZ - \delta S - \mu S^2$. Applying a similar process to the zombie and deceased classes results in our basic model

$$\begin{aligned} \dot{S} &= \pi S - \beta SZ - \delta S - \mu S^2, \\ \dot{Z} &= \beta SZ - \alpha SZ, \\ \dot{D} &= \alpha SZ + \delta S + \mu S^2. \end{aligned} \tag{1}$$

This model is based in science - as much as any zombie model can be - and thus does not allow deceased humans to reanimate. However, with a magic-based model, we can allow a curse to bring the dead back as zombies. This necessitates a fourth class: R, the resurrectable dead. Members of this class are humans who have died from any cause other than zombie attack. The dead leave this class at a rate ρ. The class D now consists solely of destroyed zombies. These changes yield the new system

$$\begin{aligned} \dot{S} &= \pi S - \beta SZ - \delta S - \mu S^2, \\ \dot{Z} &= \beta SZ - \alpha SZ + \rho R, \\ \dot{R} &= \delta S + \mu S^2 - \rho R, \\ \dot{D} &= \alpha SZ \end{aligned} \tag{2}$$

where ρR is the number of corpses that have risen from the grave to become zombies.

In the face of the end of the world, humanity would certainly fight back in every way possible, so we now add a cure into our model. We define τ as the number of days a freshly turned zombie can decompose before it cannot be cured. This necessitates a split in the zombie class, so we use Z_1 for cureable zombies and Z_2 for those who have decayed beyond repair. Additionally, the cure is not perfect - its success rate is defined as ϵ. With these modifications, the virus model is now

$$\begin{aligned}
\dot{S} &= \pi S - \beta S Z_1 - \beta S Z_2 - \delta S - \mu S^2 + \epsilon Z_1, \\
\dot{Z}_1 &= \beta S Z_1 + \beta S Z_2 - \alpha S Z_1 - \frac{1}{\tau} Z_1 - \epsilon Z_1, \\
\dot{Z}_2 &= \frac{1}{\tau} Z_1 - \alpha S Z_2, \\
\dot{D} &= \alpha Z_1 + \alpha Z_2 + \mu S^2 + \delta S,
\end{aligned} \tag{3}$$

and the curse model becomes

$$\begin{aligned}
\dot{S} &= \pi S - \beta S Z_1 - \beta S Z_2 - \delta S - \mu S^2 + \epsilon Z_1, \\
\dot{Z}_1 &= \beta S Z_1 + \beta S Z_2 - \alpha S Z_1 - \frac{1}{\tau} Z_1 - \epsilon Z_1, \\
\dot{Z}_2 &= \frac{1}{\tau} Z_1 + \rho R - \alpha S Z_2, \\
\dot{R} &= \delta S + \mu S^2 - \rho R, \\
\dot{D} &= \alpha S Z_1 + \alpha S Z_2.
\end{aligned} \tag{4}$$

3 Results

The values for π and δ were set at $\pi = 0.02062$, $\delta = 0.009$, and these do not change for any of the following results. A city of 100,000 people was chosen as the site for our zombie apocalypse, and it is assumed to be closed to the rest of the world. Additionally, we assume that through some horrible accident or experiment, half the human population in our town has been infected, so we start with $Z(0) = 50$ and $S(0) = 50$, measured in thousands of people.

We begin by examining model 1. From the equation for $\dot{Z}$, we see that the zombie population will reach equilibrium immediately, meaning $\dot{Z} = 0$ when either the entire zombie or human population is zero, or when $\beta = \alpha$. This second case has interesting results: coexistence between humans and zombies is possible for values of α and β smaller than 0.0002343. The zombie population remains constant, while the human population experiences exponential growth. This is because over time, the μS^2 term goes to zero, so we have $\dot{S} = S(\pi - \beta Z - \delta)$. As this is of the form $\dot{S} = aS$ where a is a constant, we see that $S = 50e^{\pi - \beta Z - \delta}$. This means that for $\beta > \frac{\pi - \delta}{Z}$, the human population will go to zero, and for $\beta < \frac{\pi - \delta}{Z}$ the human population grows exponentially.

If $\beta \neq \alpha$, then we assume that $\beta + \alpha$ will be fairly close to one. This assumption would be uninteresting in the $\beta = \alpha$ case, as our only choices for β and α would be too big to allow humanity's survival. With $\beta \neq \alpha$, we assume $\beta + \alpha \approx 1$ because almost every fight between a human and a zombie will end in either the destruction of the zombie or the infection of the human, but it is possible for a human to defeat a zombie while still sustaining an infection-causing wound. For $\beta > \alpha$ we again see the destruction of the human race. However, around $\alpha = 0.68$ and $\beta = 0.33$, humanity is able to overpower the zombie plague (see Figure 1). Additionally, this apocalypse spans only a few days, with the vast majority of humans dying during the first 24 hours.

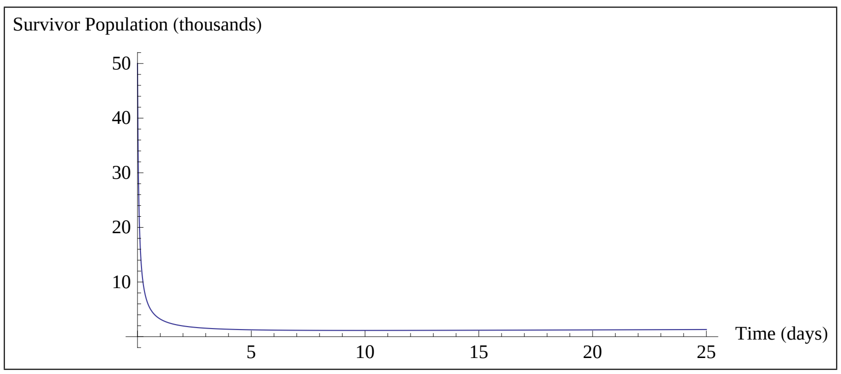

Figure 1: Basic model with $\alpha = 0.68$ and $\beta = 0.33$.

Moving on to model 2, we see that the survivor equation remains unchanged, and the zombies have gained a term that allows the population to grow independently of the surviving human population. We assume that only corpses whose time of death was after the plague began are capable of resurrection. Additionally, we assume that the number of corpses rising from the grave is proportional to the number of corpses available. We also do not allow destroyed zombies to become resurrectable corpses. Now, with the same α and β that allowed humanity to survive before, we see that even with a resurrection rate of only $\rho = 0.01$ humanity does not survive (see Figure 2); after about 150 days, humanity is wiped out, and the zombie population starts increasing noticeably at about 200 days. Increasing α to 0.7 allows humanity to survive the apocalypse and ends in the destruction of all zombies.

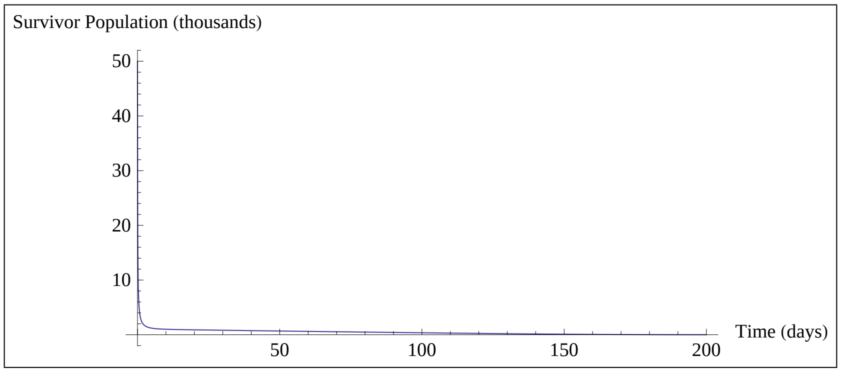

Figure 2: Curse model with $\alpha = 0.68$, $\beta = 0.33$, and $\rho = 0.01$.

We now examine model 3, the cure based in science; after all, this would probably be humanity's primary defense against the undead plague. All zombies in this model start out as freshly converted, so $Z_2(0) = 0$. We begin optimistically, assuming our cure has a success rate of $\epsilon = 0.25$. In addition, we assume that after $\tau = 15$ days, a zombie can no longer become human. We see that with the same α and β required for human survival by the first model, humanity fares much better (see Figure 3). Given this ϵ and τ, it is possible for humanity to survive even if they kill zombies much less efficiently. With $\alpha = 0.6$ and $\beta = 0.35$, the human population bounces back after less than 100 days, and the zombies are completely wiped out.

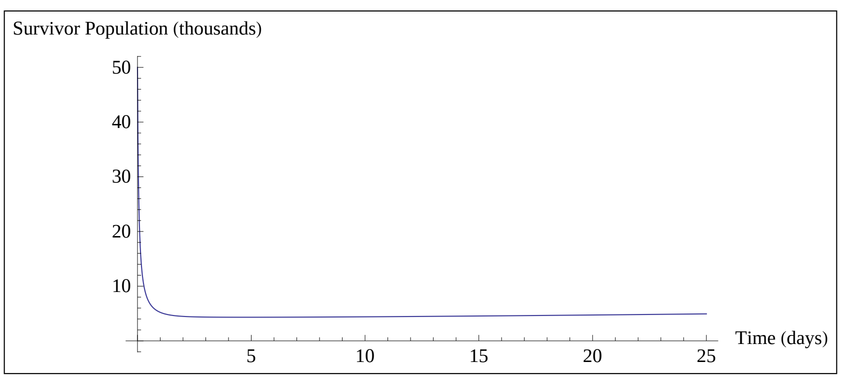

Figure 3: Science-based cure model with $\alpha = 0.68$ and $\beta = 0.33$.

Finally, we have model 4, the magic-based cure. The difference between this model and the model without a cure is as expected (see Figure 4). Humanity fares much better with the addition of a cure, and they are able to kill zombies much less effectively without causing the end of humanity. Even with a cure success rate of only 2%, the survivor population avoids extinction under conditions that had previously destroyed it, while completely eliminating the zombie threat.

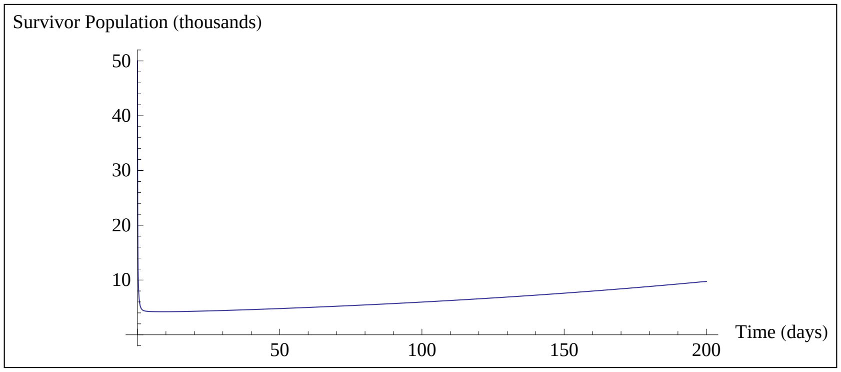

Figure 4: Magic-based cure model with $\alpha = 0.68$ and $\beta = 0.33$.

4 Conclusion and Directions for Further Research

Future paths for this research are numerous. This model does not examine the effects of most human behaviors and simply assumes that every time a zombie and a person meet, there will be a fight to the death. Including quarantine and humans running from zombies rather than fighting them could have interesting effects on the zombie and survivor populations. This model also assumes that the zombie plague causes a human to become a zombie almost immediately after infection. The effects of an incubation period would be devastating, as it would allow humans hiding the disease to enter survivor safeholds, turn, and wreak havoc.

These models have some fairly heavy implications for humanity in case of zombie apocalypse. First, it is very difficult for the human race to survive if people have less than a 70% kill rate when fighting zombies and if the zombies infect people in more than 30% of the fights. These rates may be somewhat unreasonable given that average person has little to no combat training. Second, given the results of this paper, it appears that the best course of action would be to work on a cure for zombism as quickly as possible. There is some good news: regardless of the outcome, at least it will all be over quickly.

References

[1] Max Brooks. *The Zombie Survival Guide: Complete Protection from the Living Dead*. Three Rivers Press, 2003.

[2] A. George. *Epic of Gilgamesh*. Penguin Classics, New York, 2003.

[3] P. Munz; I. Hudea; J. Imad; R.J. Smith?[sic];. When zombies attack!: Mathematical modelling of an outbreak of zombie infection. *Infectious Disease Modelling Research Progress*, pages 133–150, 2009.